Quantum electrodynamics (QED) contributions to the electron $g$ factor in strong binding fields have been tested recently to high precision in Penning trap measurements: an experiment with $^{28}\text{Si}^{13+}$ allowed to benchmark certain higher-order QED corrections for the first time [1], and constitutes one of the most stringent benchmarks of strong-field QED theory. Recently, the uncertainty of the electron mass has been largely decreased via measurements on the $^{12}\text{C}^{5+}$ ion, and by using the theoretical value of the $g$ factor [2, 3]. In order to further reduce uncertainties in the theoretical description, we calculate further higher-order corrections, such as the higher-order remainder in $Z\alpha$ for the one-loop self-energy corrections [4], and parts of the two-loop Feynman diagrams [5, 6].

An independent and improved determination of the fine-structure constant $\alpha$ may also be possible in near future employing a weighted difference of the $g$ factors of the H- and Li-like ions of the same element. This weighted difference is chosen to maximize the cancellation of detrimental nuclear effects between the two charge states. It is shown that this method can be used to extract a value for $\alpha$ from bound-electron $g$-factor experiments with an accuracy competitive with or better than the present literature value [7]. We anticipate that the necessary theoretical accuracy can be reached by a combination of non-relativistic QED methods, and QED in the Furry picture ($1/Z$ expansion) [8].